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BASIS OF THE TERMINOLOGY AND ALGORITHM FOR THE SOLUTION OF INVERSE HEAT-TRANSFER PROBLEMS

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The terminology of thermal engineering, and particularly the phenomenological theory of heat conduction in which inverse heat-conduction problems enter, lags behind the level of development of this intensively developing branch of science. The lag caused by the accelerated development of the class of inverse problems is that many problems and methods for their solution have no rigorous scientific description in clearly formulated criteria and have not appropriate clear definitions, terminological designations. By definition, terminology is a word or word-combination called upon to denote a concept and its relationship to other concepts exactly within the limits of a special sphere. Ideal terminology should be unique, systematic, and stylistically neutral.

Let us first of all separate the inverse heat-transfer problems into several already existing groups according to the criterion, the kind of heat transfer. Inverse problems (IP) which are solved or can be solved for technological processes and (or) technical systems for which the thermal operational aspects are investigated can be called heat exchange, heat transfer, heat transmission IP or thermal IP. Depending on the kind of heat exchange, these thermal IP can be separated into IP of heat conduction, convection, radiation, and finally, IP of complex convection—radiation—conduction heat transfer. This latter class of IP is substantially the thermal IP or heat transfer IP if all kinds of heat exchange are understood to be heat transfer.

Usually, and moreover, habitually, the terminology "heat conduction IP" is used although it is often a question of more complex (in the kind of heat exchange) heat transmission IP. Below, as a rule we speak of inverse heat-conduction problems (IHCP) and the abbreviation IHCP refers to inverse heat-conduction problems and not to IP of heat exchange, generally.

Heat exchange IP and IHCP, particularly methods of their solution, have been the subject of hundreds of journal papers and tens of monographs* (the books [1-7] have been devoted to IHCP, for instance), but up to now there has been no classification of the problems and methods corresponding to the requirements imposed on scientific classifications [8], despite the numerous (sometimes contradictory) proposals [1-7].

*We have in mind not only monographs devoted to heat transmission IP but also numerous monographs on methods of solving IP in different branches of science in which the mathematical models are isomorphic with the mathematical models of heat exchange.

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Much of the terminology, the designation of problems (DP) and inverse problems (IP), does not correspond to the requirements presented above insofar as information retrieval of the research is practically impossible. For instance, the whole set of types of problems entering into inverse problems are called inverse heat-conduction problems, and simultaneously, separate types of problems are called "inverse." For example, external inverse problems in which we find the boundary conditions are called simply "inverse," "boundary inverse." Inverse problems in which we find coefficients or terms entering into the main equation, the heat-conduction equation, are called "coefficient" equations although there are coefficients in the boundary conditions and in the initial condition in the mathematical model of the heat-conduction phenomenon. Many authors who propose or use the classification of problems and (or) methods of solving problems (inverse heat-conduction problems (IHCP) in our case) do not mention the classification criteria, which results in an incorrect appraisal of the research, and consequently, inadequate utilization of the results of scientific developments in theoretical and applied investigations.

It is emphasized in [8] that "the abundance and poor order in the new concepts and terminology, printed and unpublished material makes difficult the retrieval and utilization of necessary data, which causes and information deficit that retards social progress. Development of an optimal classification consequently becomes not only a scientific but also an economically important problem" [8, p. 269]. Papers on terminology sometimes seem to be less informative than research devoted to specific methods of solution and results of solving IP. Many years of research on the primary sources of IP and methods of solving them, retrieval of the necessary method of scientific investigation in the literature [1-7, 9], and, finally, the program paper [10] obliged still another recollection of the problems of terminology and classification of IP and methods for their solution — once more, because a classification of IP and methods for their solution is proposed in the papers [11, 12], in the report [13], and in the monograph [7]. The IP classification [7, 11, 13] is constructed by deduction [8], is natural, which means 1) general initial concepts are given and substantial criteria of the objects being classified, the problems and their methods of solution are taken as the foundation of the classification. Precisely such a natural classification "can be the source of knowledge of the objects being classified" [8, p. 269].

There is a standard natural classification of problems according to the kinds of heat exchange in the theory of heat and mass transfer: 1) heat conduction; 2) convection; 3) radiation; 4) heat exchange in chemical and phase transformations.

Problems classified according to the kinds of heat exchange can be classified according to many other criteria, for instance, according to the dependence of the desired quantities on time (stationary, nonstationary), coordinates, uniformity of the bodies being investigated, etc. An example of such a classification of direct heat-conduction problems is presented in [14]. Up to now inverse heat-conduction problems have no generally applicable and steady classification although first episodically and then systematically they were resolved more than 25 years ago in thermal engineering practice. The paper is not devoted to the history of the question. It is possible to refer to certain historical information available in monographs [1-7, 14, 15] to emphasize that proposals to classify both the problems and the methods for their solution are contained in each of these books. Before turning to the classification, we note that classification is inseparably related to terminology since the classes isolated require designation, a new term. The terms that were ascribed to new problems and methods of solution did not often correspond with the requirements listed at the beginning of the paper. For example, the term "direct methods" of solving inverse problems (IP) is not defined in [16] and does not correspond to the definitions of the term "direct methods" in [17]. In every case, the criterion by which direct methods are distinguished from indirect is mentioned in [16], while it follows from the content of [16] and [17] that the criterion indicated in [17] differs from the criterion by which the author of [18] separated the methods into direct and, evidently, variational (pp. 531, 533 in [18]). A definition of direct methods is given in [18] which does not correspond to the definition in [17]. Therefore, the identical term is applies to denote different concepts. The designation "inversion method," "inverse correspondence method" is suitable for the methods called "direct" by the definition of the author [16, 18, p. 531]. We speak about this designation below.

Thus, a new class of problems in mathematical physics was extracted, particularly, in thermal engineering, which was called "inverse problems."

By what criterion are all problems (including the heat-conduction problems) divided into direct and inverse? By the criterion of that which is desired, the cause or the effect. In direct problems the effect is determined from the causes. In inverse problems, causes are found in terms of effects. Thus, division of mathematical physics problems into direct and inverse presumes knowledge of the cause—effect relationship. It is shown in [9] that the definition presented is not unique, but in the author's opinion [19] (and in our's) is uniquely correct.

Let us consider the criteria by which inverse heat-conduction problems can be classified, and let us consider certain designations, terms, which are proposed in different publications as applicable for the description of different types (classes) of IP. The first criterion is: the aspect of the physical phenomenon that is estimated in solving the IP, qualitative or quantitative. When the qualitative side is estimated, we seek the form of the mathematical model of the phenomenon being investigated. Such problems are called structural identification problems generally, and inductive problems in the heat-conduction theory in particular. Let us recall that the term "identification" in the most general sense of this word denotes "oneness, equivalence, sameness," while in science and engineering it denotes the selection of the form and content of the mathematical description (mathematical model). This model is optimal in some sense (the optimality criterion should be chosen or given) and is constructed by means of realizations of the input and output signals of the object being identified. Thus, a qualitative inverse problem is a structural identification problem. In the heat-conduction theory where the model is more or less known, the form of the model is just refined and the qualitative problem is called an inductive problem [14, 15].

Problems of searching for quantitative values of the coefficients or terms of the mathematical model will be called "quantitative." These problems are the subject of parametric identification and depending on the site of the desired quantity in the mathematical model are called external inverse, inverse (internal inverse), inverted in heat-conduction theory (Fig. 1).

The second criterion for the classification of quantitative inverse problems is the place of the desired quantity in the mathematical model of the thermal phenomenon being investigated. The mathematical model consists of the fundamental equation (the heat-conduction equation in our case) and the boundary conditions — the conditions of single-valuedness in which the boundary conditions and the initial condition enter.

The problem is called external inverse (boundary*) when according to the temperature effects (found in experiment T_e) we seek the boundary conditions of the I, II, III, and IV kind. Correspondingly, the external inverse problems can be called external inverse problems of the kind I (search for T_s), II (search for q_s), III (search for α or T_m), and IV (search for T_s and q_s simultaneously).

Methodologically it is interesting to stress the following. Just one classification criterion, the kind of boundary conditions, was chosen above. The classification was then made according to this criterion.

The problem is called inverse (internal inverse, coefficient) when we seek the coefficients or terms inside the fundamental transport equation (heat conduction equation) by means of the effects T. Such coefficients are the coefficient of heat conduction (λ), the bulk specific heat ($c_V = c_\rho$), and the power of the bulk internal heat sources (q_V). The search for λ and c_V can be called the inverse problem of the kind I, and the search for q_V the inverse problem of the kind II.

The problem is called inverted (retrospective, time) when we seek the previous (often initial) temperature distribution according to the effects T.

The object of identification can be selected as a classification criterion. The inverse problem will then be solved for a shell, a vessel, a support, a device, etc. The heat-transfer process can be selected as a criterion. Then the inverse problem (IP) will be an IP of heat conduction (IHCP), convection, radiation. An individual subject or a system of subjects related by thermal bonds can be selected as a criterion. Then the IP can be called an IP of a specific element and of heat transfer in a technical system.

*As a rule, we present the terms used by other authors in parenthesis.

Classification criteria	Inverse problems								
Criterion I, aspect of the phenomenon	Quantitative (parametric identification)								Qualitative (structural identification)
Criterion II, part of the mathematical model (MM) in which the desired quantity enters	Boundary conditions				Fundamental transport equations		Initial conditions		Mathematical model of the first kind
Problem designation	Inverse, external inverse				Inverse, internal inverse		Inverted (retrospective)		Inductive
Genus of problem	I	II	III	IV	I	II	I	II	—
Criterion III, desired quantities	T_s x_{is} τ	q_s x_{is} τ	α, T_m x_{is}	T_s, q_s ΔT_c, q_c x_{ic}, τ	λ, c_V x_i in τ	q_v, w_i x_i in τ	$T(x, 0)$ τ	$T(x, \tau_{n-1})$ τ	Form of the mathematical model
Criterion III, generalized designation of the desired quantities	Coefficient				Coordinates		Time		Preceding temperature field
Problem designation	Coefficient				Geometric		Time		Retrospective
Criterion IV, desired quantities	T_s, q_s, α, T_c, λ, c_V, q_v, ΔT_c, q_c, w_i, $T(x_i, 0)$ $T(x_i, \tau_{n-1})$				$x_{i\,in}$, x_{is} $i=1, 2, 3$		τ $\tau = n_j\delta\tau_j$, $j = 1, 2, \ldots$		$T(x, 0)$, $T(x, \tau_{n-1})$

Fig. 1. Classification of IP according to different criteria: $i = 1, 2, 3$ for three directions of any orthogonal coordinate system; $n = 1, 2, \ldots$ is the number of instants $\tau = n_j\delta\tau_j$; $j = 1, 2, \ldots$.

But it is impossible to divide problems in different criteria into classes as is done in [5] (p. 10). It is impossible to divide problems into inverse heat-conduction problems and inverse problems of heat transfer in technical systems. The former are isolated according to the criterion of the kind of heat transfer. The latter are isolated according to the content of the object without any indication of the kind of heat transfer therein.

In the former, heat-conduction problems are solved in both systems and objects, while in the latter, heat transfer in technical systems occurs by heat conduction, convection, and radiation. Inverse heat-conduction problems are a particular case of heat-transfer problems in technical systems and are extracted from them according to the criteria: a) the kind of heat transfer; and b) the degree of hierarchy of the object under investigation in the given system.

The fact should be noted that different mathematical models are identified according to a given classification contradiction: a heat-conduction model with its boundary conditions in the IHPC, and energy conservation equations written for all the elements and the intermediate media (between the elements) of the system in the heat-transfer IP in technical systems. As a rule, these equations are written in balanced form: the system of such equations takes account of all the thermal bonds and all kinds of heat transfer (including heat conduction).

It is interesting to note that the methods of the theory of identification were applied to solve the inverse problems of heat transfer for technical systems in [20], and its author considers the system of balance equations a certain approximation of the heat-conduction equation with equivalent heat-transfer coefficients to take account of convection and radiation. This manner of utilizing statistical estimation methods was later developed successfully in application to inverse problems for heat-conduction processes in [3].

Let us turn to the IHCP. The IHCP can be classified according to the designations of any of its terms in the mathematical model. All the quantities except the temperature T, the effect, are causes in the mathematical heat-conduction model. Let us enumerate them: λ, c_V, q_V, x_i, τ, q_s, T_s, α, T_m, $T(x_i, 0)$, etc. In every specific case, the quantities desired in the IHCP can enter into q_V and q_s. For instance, if $q_s = \varepsilon\sigma(T_s^4 - T_m^4)$, then ε, the absorption coefficient of the surface subjected to radiation, can be sought by means of T_e. The quantities mentioned above are the coefficients, the coordinates (describe the form), and the time (τ). It is logical to call the IHCP coefficient, geometric (or coordinate), and time problems, depending on the designation of the desired quantity.

Starting from this classification criterion, coefficient inverse problems are all IP in which we seek the coefficients, and not only those problems in which the coefficients λ, c_V are determined from the fundamental heat-conduction equation. It is characteristic that inverted (retrospective) problems are coefficient problems according to the proposed classification (Fig. 1), since the desired initial distribution (or any previous distribution) can be a coefficient in the initial mathematical heat-conduction model.

Inverted problems are therefore extracted into a special kind of criterial problem, the part of the MM in which the desired quantity enters. Moreover, they are extracted according to the criterion of the desired quantity in the coefficient IP, and according to the specific designation of the desired quantity, the previous potential field, are called "retrospective" or "inverted" as is customary.

In our opinion, such a classification of problems, the considered criteria, and the proposed designations correspond to the requirements on the terminology and classification described above.

Before turning to a classification of methods of solving IP, it is necessary to examine the interrelationship between the IP and the problem of optimal control of thermal processes (OCTPP).

The OCTPP differ from the IP of heat transfer in that the optimal temperature distributions in the bodies are initial and given, rather than the experimental temperatures T_e (i.e., taking place in a real physical process). The optimal temperature distribution is given, but it can be such that under given conditions it is impossible to realize in this problem under any controls (conditions). The class OCTPP in even such a limited formulation — the optimal temperature distribution is given — is broader than the class of heat transfer IP. As a rule, it is considered in the heat-transfer IP that the problem of controllability (the problem of existence of a solution) is solved, but is not in the OCTPP. Moreover, the OCTPP can be formulated as a problem to search for the optimal controls to assure other optimal indices (criteria) of the thermal process, i.e., not the optimal temperature distributions but other criteria, for instance, the thickness of the thermal insulation, thermal strains, times to achieve given temperatures or strains, etc.

In the general case, the OCTPP is a problem in thermal projection (in the terminology of the author of [5]). In engineering practice, projection of computations in the solution of such problems had and has the designation "structural analyses," while the temperature field computations in direct problems are called "verifying analyses."

Engineers in designing articles and structures in structural analyses solved those problems of searching for causes by means of effects, which have only been called "inverse problems" generally, "inverse heat-conduction problems," "inverse problems of heat transfer or heat transmission" in the last 15-20 years, particularly for the problems we have under consideration that are associated with complex convective—radiation—conductive heat exchange in thermal systems.

Let us examine the method of solving the IP of heat transfer. All the common methodological considerations associated with problem classification and with terms—designations refer completely to the methods. The methods as well as the problems should be considered as a system of coordinated concepts that should be classified according to substantial criteria so (as has already been noted above) that the classification would be natural, scientific, and would be a source of knowledge about the objects being classified. It is first necessary to define the term "method." As a rule, the method of solving a problem is, as a rule, understood to be the set of operations, recipes, actions that permit achieving the target set. Our purpose is to obtain numerical values of the quantities desired (causes) by having a mathematical model of the inverse problem and experimental temperatures (T_e) (effects). The set of operations, recipes, actions which is performed in a definite sequence (the set of sequential stages) to achieve this target is called "the method of solving IP." From such a definition there follows that we call the method of solving the IP that which is called the "algorithm of the solution" in cybernetics and in computer mathematics.

Essential in such a definition is the accent on the fact that the method of solving IP is a set of actions, and for each action—recipe its own method can be used as the method of solving that individual problem that is solved at a given stage. One of the possible schemes for the algorithm to solve the IP is presented in [12, 13]. The diagram is presented in Fig. 2 with an indication of the recipe-stages for solving the IP. The whole algorithm can be called general, or a "general method" for solving IP. Certain particular methods of solving problems and methods—organizers are used in each of the stages.

The general method of solving IP is often designated by the method* applied to one of the stages of the general algorithm. Application of designations that methods of solving direct problems have, or designations that methods of solving particular problems in individual stages of the general method or the general algorithm have, for the general methods of solving IP produces a terminological maze. We spoke above of the direct methods that are applied in different divisions of mathematics for the solution of DP. The very same can be said about variational methods.

Let us examine the criteria by which the general methods of solving IP can be classified, without considering the classification of methods used in each stage-recipe of the general algorithm (Fig. 2) in detail. The fundamental criterion is the presence of the operation of searching for the extremum of the residual functional $\Phi(\varepsilon)$ in the algorithm.

When the experimental temperatures are substituted directly into the mathematical model of the I or II kind, we have model inversion methods or (by the traditional designation)

*For example, an iteration method to determine T or an iteration method of minimization; the method of finite differences to search for T, the method of least squares to identify the coefficients of a given dependence of the quantity desired, etc.

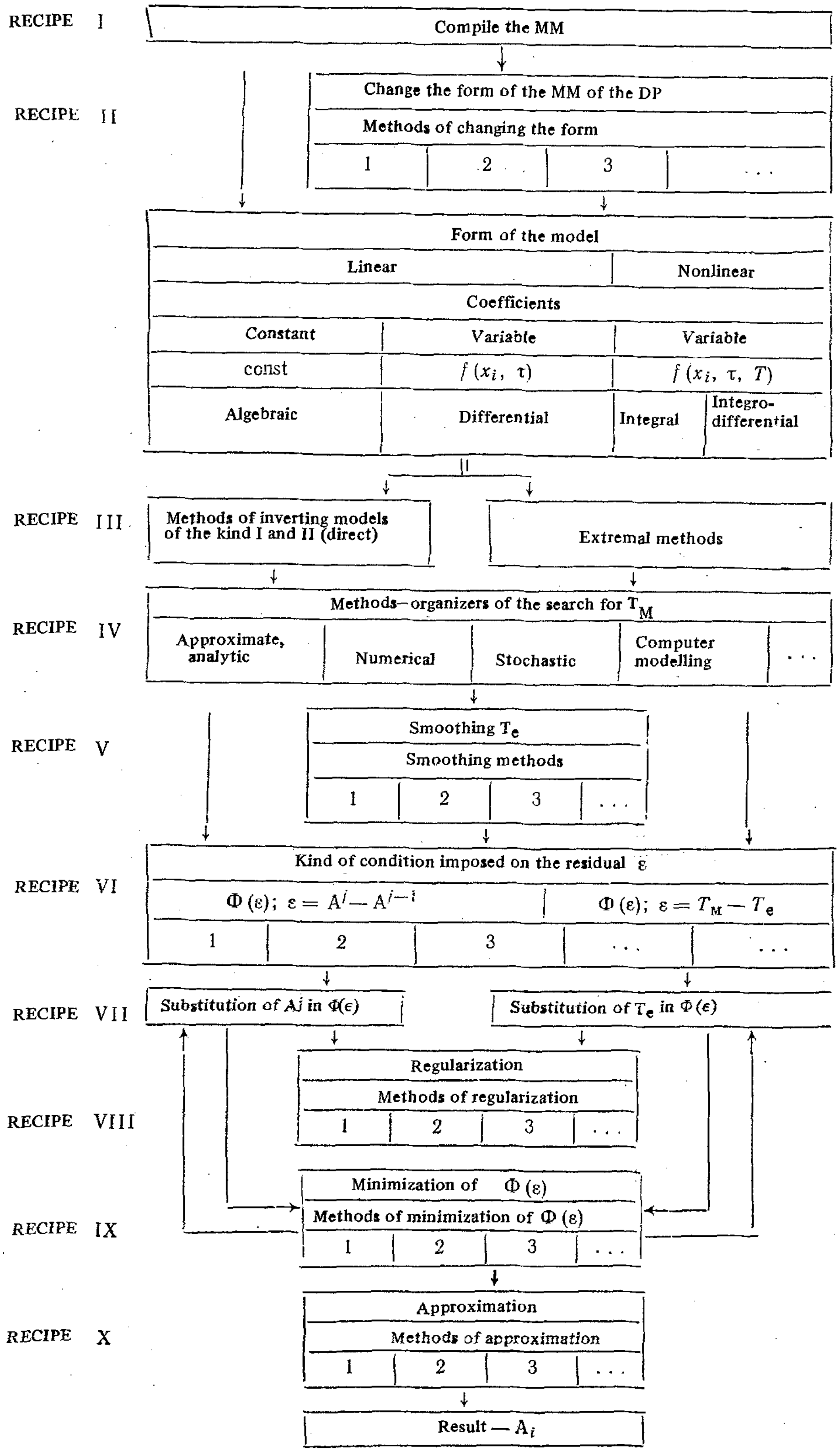

Fig. 2. Diagram of the algorithm to solve IP by different methods.

direct methods. The experimental temperatures T_e are substituted directly (direct methods) into the model. The mathematical model of the direct problem is inverted here.

From the model $T_M = f(A)$ we have $A = f^{-}(T_e)$, where T_M is the temperature of the model, f is the direct operator, f^{-} is the inverse operator (the operation of inversion), A is the desired cause, T is the effect, and e is experimental.

We call the initial DP model the mathematical model of the first kind. This model is transformed, changed by special methods, recipes, procedures (see recipe II, Fig. 2). This changed model, for instance, an integral equation, a system of algebraic equations (finite differences), a system of ordinary differential equations (direct method), can be inverted. To invert means to write in such a form that A could be extracted from the model of the kind I (for example, if $T = L(a)$, then the inversion operation (operator) is L^{-} and $a = L^{-}(T_e)$).

We call the solution, i.e., the expression obtained as a result of solving the equations entering into a model of the kind I, a mathematical model of the kind II. This solution can be inverted, i.e., an expression can be obtained for the search for A. For instance, a solution of the DP by any method of solving the heat-conduction problem is obtained. From this solution we obtain a computational formula to determine A (the heat-exchange coefficient or the thermal diffusivity coefficient, etc.) by the method of integral transforms or by the method of self-similar solutions. All the nonstationary methods of determining the thermophysical properties of substances (methods of a regular regime, methods of "flashes," etc.) are constructed by the procedure of inversion of models of the kind I or II.

Thus, the first group of methods is methods of inverting models or direct methods. There is no procedure for searching for the extremum of the residual functional $\Phi(\varepsilon)$ (the residual is $\varepsilon = T_M - T_e$). The temperatures T_e are substituted into the inverted model of the problem and data about A are extracted.

The second group of methods contains the operation of searching for an extremum of the residual functional $\Phi(\varepsilon)$. A certain expression is ordinarily minimized – the criterion-functional in which the model T_M and experimental T_e temperatures are compared. The causes A are considered to satisfy the mathematical model for the search for T_M if an extremum of the residual functional, deviations of ε, exists. Therefore, control of the solution is by the main law of process control – by the law of controlling the deviation – in methods called extremal.

In the author's papers ([5], say) extremal methods are called variational. But the recipe IV (Fig. 2), the recipe for searching for model T_M, always takes part in extremal methods. Variational methods of solving direct problems are applied for their search. It is rational to call methods of solving IP not by the designations of the methods for solving DP, which we apply in one of the stages of the general algorithm for the solution of IP, and not by methods-organizers, but to ascribe a designation (let it be new) to the whole algorithm.*

Let us examine the general methods of solving IP. Some of the inversion methods (direct methods) and some of the extremal methods already have a common designation for the whole algorithm. These designations could evidently be retained: for instance, the method of successive intervals, methods of partial and coordinate functions, gradient methods, etc. Incidentally, not only methods of searching for the minimum of a function are called "gradient" methods, but also methods of inverting the solutions of IP used in thermal engineering to solve external inverse problems [21].

We consider briefly the classification of extremal methods which is unsatisfactory, according to our requirements, for scientific classification. Extremal methods can be separated into four groups: selection methods (samples), minimization, identification, and methods of optimal control theory. Each of the listed groups contains several (sometimes tens of) methods for searching for the extremum of the residual functionals. But there is no clear criterion for the separation, at least because the breakdown into identification methods and optimal theory methods is made according to a functional criterion (i.e., according to the branch of science in which the criterion is used). The very same minimization methods of the same residual functionals are often applied in the identification and optimal control theories.

Methods containing a regularization procedure, for instance, regularizing algorithm–methods of solving IP, could be extracted. Indeed, regularization is one of the strong

*The term "direct methods" is not faultless in this respect.

recipes used in solving IP but inverse problems can also be solved without regularization. However, when applying direct methods the incorrectness of the inverse problems according to Hadamard (the instability property) is felt much more strongly than in the regularizing algorithms of extremal methods. The value of the scientific direction in the development of methods of solving IP, which was started by A. N. Tikhonov, is not in the introduction of regularization as an always effective method for solving IP, but in the radical methodological revolution generated by the change in the traditional representation of the fact that it is impossible either to pose or solve incorrect problems according to Hadamard. By introducing the concept of "regularization of the solution of an incorrect problem," A. N. Tikhonov transferred the problem from the plane of the "pointless and useless" searches to the plane of the "development and perfection of specific algorithms" [19, p. 105]. The gnosiological role of regularization methods should be recalled precisely thus, and extremal methods of solving IP with regularization should rightly be called regularization methods or A. N. Tikhonov methods.

In conclusion, the principal aim which the author has posed should be stressed: a clear classification of problems and methods for their solution will accelerate many times the information search for necessary methods to solve important scientific—technical problems. It is impossible to accept the situation that it is more rapid and cheap to solve a problem already solved then to find the source where the solution of this problem is presented. Development of a new method or solution of a new problem is certainly more prestigious from the viewpoint of the individual researcher than classification and a fast informational search, but the latter may induce a greater effect in both theoretical and applied effects from the aspect of a collective of researchers.

NOTATION

T, temperature; A, cause; f, direct operator; $\bar{f}$, inverse operator; λ, heat-conduction coefficient; c_V, specific heat; q, thermal flux intensity; α, heat-transfer coefficient; τ, time; x_i, coordinates (i = 1, 2, 3); ε, coefficient of absorption; σ, Stefan—Boltzmann constant. Subscripts: e, experimental; M, model; s, surface; m, medium; v, V, volume; c, contact; in, internal.

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MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER IN FILM CONDENSATION

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The state of the problem of mathematical modeling of the heat and mass transfer during film condensation of a gas is considered.

Heat-exchange apparatus intended to transfer the substance from the gaseous to the liquid state — condensers — started to become widespread at the end of the 19th and beginning of the 20th centuries in connection with the appearance of steam-condensation turbines. At this time the thermal power of powerplant condensers is about 4 TW in the world. As a rule, the process of film condensation of vapor is realized in this apparatus.

The first theoretical work on heat exchange with condensation (W. Nusselt) appeared in 1916 [1], its results were the basis of methods of designing industrial condensators for decades. The majority of investigations on condensation reduced substantially to determining corrections to the Nusselt formulat to compute the heat-transfer coefficient. In 1954 Chernyi [2] published a paper which set the beginning of a qualitatively new stage in the development of a theory of condensation. The author of this paper first represented the model of film condensation in the form of two conjugate boundary layers (one of which is the condensate film), which permitted application of the well-developed apparatus of boundary-layer theory to investigate this process. This approach was utilized later to solve more complex condensation problems: vapor—gas systems [3, 4], a chemically reacting gas [5], etc.

Two categories of problems on film condensation exist: external and internal. For the external problems the gas stream parameters outside the boundary layer limits (temperature, velocity, composition in the case of multicomponent gas condensation) remain constant. In industrial apparatus with condensation on the outer surfaces, the film flow regime is kept mainly laminar or wave laminar.

The gas flow parameters vary continuously along the condensation surface during condensation inside tubes and channels, and the most diverse combinations of gas and liquid flow regimes can take place in the very same apparatus. In the general case, laminar film flow

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